A MODEL OF AN ARC IN A TRANSVERSE GAS FLOW

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The analytical results of [4] concerning velocity and enthalpy of a gas near an arc are refined. The arc radius and enthalpy distribution over the arc column section are calculated.

The few analytical studies that have been performed of the state of a gas in an electric arc discharge have mainly dealt with longitudinal gas draft along the arc [1-3]. The present study will offer results of a theoretical investigation of transverse gas draft across an arc, the current–voltage characteristic (CVC) of which was obtained in [4] for the case of small change in gas velocity and enthalpy near the arc column.

We will now define the gas flow parameters beyond the arc more accurately. The system of equations describing the gas flow has the form

$$\frac{\partial h}{\partial x} = \frac{\partial}{\partial \varphi}\left(\nu_0 v_x \frac{\partial h}{\partial \varphi}\right), \tag{1}$$

$$\frac{\partial v_x}{\partial x} = \frac{\partial}{\partial \varphi}\left(\nu_0 v_x \frac{\partial v_x}{\partial \varphi}\right), \tag{2}$$

$$\rho h = \text{const}, \quad \nu\rho/h = \text{const}, \quad \mathrm{Pr} = 1.$$

We require that the solution of Eqs. (1), (2) satisfy the relationship $v_x/h = v_0/h_0$ and the following boundary conditions:

$$v_x(\infty) = v_0, \quad v_y(\infty) = 0, \quad h(\infty) = h_0, \quad \rho(\infty) = \rho_0, \quad h(r_1,\ 0) = h_1,$$
$$(\partial v_x/\partial y)_{y=0} = 0, \quad (\partial h/dy)_{y=0} = 0.$$

We divide the region beyond the arc into two sections. In the first, closest to the arc, $v_x \gg v_0$ and $h \gg h_0$. In the second section, as in [4], the gas velocity and enthalpy differ little from their values at infinity and are described by expressions identical in form

$$h = h_0 + \frac{B}{\sqrt{x}} \exp\left(-\frac{\varphi^2}{4\nu_0 v_0 x}\right). \tag{3}$$

Introducing the variable $\zeta = \varphi x^{-1/3}$, we find the self-similar solution of system (1), (2) for the first section:

$$h = \frac{h_0}{6 v_0 \nu_0 \sqrt[3]{x}} (G - \zeta^2). \tag{4}$$

The integration constant G and the length of the first section x* are determined from the conditions of merger of the expressions for enthalpy at the edge of the arc ($x = r_1$, $y = 0$, $h = h_1$) and the boundary between the first and second sections ($x = x^*$, $y = 0$, $h \approx 2h_0$). As a result, we obtain

$$h = \frac{h_0}{6\nu_0 \sqrt[3]{x}} \left(\frac{6\nu_0 \sqrt[3]{r_1}\, h_1}{h_0} - \frac{y^2 v_0}{\sqrt[3]{x^2}}\right), \tag{5}$$

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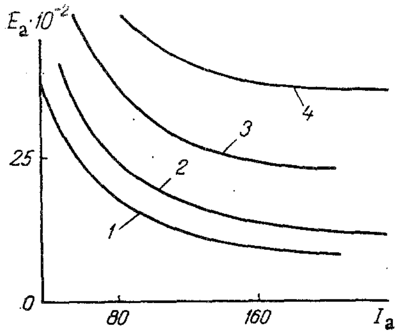

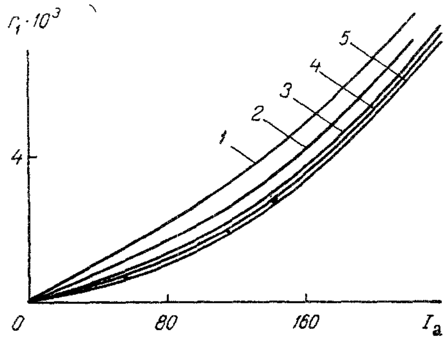

Fig. 1 Fig. 2

Fig. 1. Field intensity E_a (V/m) vs current I_a (A) and velocity v_0 (m/sec) of incident flow: 1) v_0 = 1.7; 2) 3; 3) 6; 4) 10 m/sec.

Fig. 2. Arc radius r_1 (m) vs current I_a and gas velocity: 1-4) see Fig. 1; 5) v_0 = 20 m/sec.

$$x^* = r_1 \left(\frac{h_1}{2h_0} \right)^3. \tag{6}$$

We will consider solution (5) only within the limits of a layer with outer boundary y*, corresponding to v_0 and h_0:

$$y^* = \pm \sqrt{\frac{6\nu_0}{v_0} \left(\frac{\sqrt[3]{r}\, h_1}{h_0} \sqrt[3]{x^2} - x \right)}.$$

The thickness y** of the boundary layer in the second section is defined by the condition $y^* \approx y^{**}$ at $x = x^*$. It is evident from Eqs. (3) and (5) that the indicated equality is achieved when the exponent in Eq. (3) is close to two. Hence the thickness of the boundary layer in the second section is equal to

$$y^{**} = \pm \sqrt{\frac{8\nu_0 x}{v_0}}.$$

Equation (6) permits refinement of the integration constant B in Eq. (3):

$$B = \sqrt{\frac{r_1 h_1^3}{8h_0}},$$

required for derivation of the arc CVC.

Assuming sinusoidal change in the electric field intensity in the arc, the coefficient α^2 appearing in D [4] is equal to

$$\alpha^2 = \frac{1}{E_a} \sqrt{\frac{2\omega \overline{M}}{\pi \overline{\sigma}}}.$$

Then, without consideration of radiation, the arc CVC takes on the form

$$I_a = \frac{(D v_0)^{0,5}}{E_a^{0,5} (E_a^2 - E_{min}^2)^{0,25}}, \tag{7}$$

where $D = \pi^{1,5} \rho_0^2 \nu_0 \mu_1 \overline{N}^{0,5} h_1^3 / 2^{1,5} (\omega \overline{M})^{0,5} h_0$

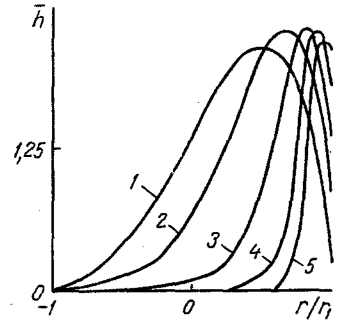

Fig. 3

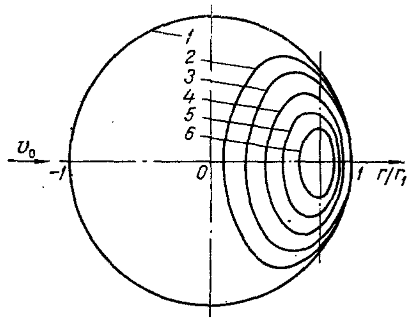

Fig. 4

Fig. 3. Gas enthalpy distribution $\bar{h}$ in arc column for various velocities: 1–5, see Figs. 1, 2; $r_1 = 4 \cdot 10^{-3}$ m.

Fig. 4. Isoenthalpy of arc with transverse draft: 1) $\bar{h} = 0$; 2) 0.1; 3) 0.2; 4) 0.4; 5) 0.6; 6) 0.8 $\bar{h}_{max}$; $r_1 = 4 \cdot 10^{-3}$ m; $v_0 =$ 6 m/sec.

Results of calculating the CVC of an ac arc in a transverse air flow for $\rho_0 = 1.2$ kg/m³, v_0 $v_0 = 1.6 \cdot 10^{-5}$ m²/sec, $h_0 = 3 \cdot 10^5$ J/kg, $\delta = 0.0075$, $h_1 = 1.5 \cdot 10^7$ J/kg ($T_1 = 6000$°K), $\bar{N} = 2600$ W/m, $\sigma = 1300$ $\Omega \cdot m^{-1}$, $\bar{M} = 1.4 \cdot 10^5$ J/m³, $\omega = 314$ sec^{-1} are shown in Fig. 1. With increase in gas velocity the electric field intensity increases, as was noted in [4, 5] for arcs with transverse draft.

We will find the dependence of arc radius r_1 on gas flow velocity v_0 and current strength I_a. Using the equation of the CVC, Eq. (7), and Eq. (4) from [4], we obtain

$$r_1 = \sqrt{\frac{KF}{2} + \left[\left(\frac{KF}{2}\right)^2 + \frac{KL}{v_0^2}\right]^{0,5}}, \qquad (8)$$

where $K = I_a^4 \mu_1^2 \bar{N}/D^2\bar{\sigma}$, $L = \mu_1^2 \bar{N}/\bar{\sigma}$.

It is evident from Eq. (8) that $r_1 \to (KF)^{1/2}$ with increase in v_0. This result is illustrated by Fig. 2.

The gas enthalpy within the arc column was calculated with Eq. (3) from [4]:

$$\bar{h} = AJ_0\left(\mu_1 \frac{r}{r_1}\right) \exp\left(\frac{wr}{2a^2}\right),$$

where

$$A = \frac{(v_0 \nu_0 r_1)^{0,5} \rho_0 h_1^{1,5}}{2^{1,5} \pi^{0,5} \bar{N} \mu_1 J_1(\mu_1) I_0(wr_1/2a^2) h_0^{0,5}}.$$

The calculated data shown in Figs. 3, 4 indicate that for r_1 = const with increase in flow velocity the maximum enthalpy values first increase, then decrease. Moreover, there is a shift in the enthalpy maxima in the direction of the flow together with deformation of the isoenthal›ies, which take on the form of ellipses for high velocities. Since the current passes ma.nly through the hottest regions of the arc column, it can easily be seen that with increase in velocity the dimensions of these regions decrease. Thus the region bounded by $h = 0.1\bar{h}_{max}$, passes 95.2% of the arc current at v = 10 m/sec, although the area of this region is only 14.7% of the arc column cross section.

Ellipse like arc column forms with major axis oriented across the gas flow were noted in [6, 7].

NOTATION

h, gas enthalpy; x, y, coordinates; φ, flow function; v, velocity; ν, kinematic viscosity coefficient; ρ, density; Pr, Prandtl number; r, radius; ω, angular frequency; $\bar{M}$, $\bar{\sigma}$, $\bar{N}$, linear approximation coefficients for density, electrical conductivity, and thermal conductivity; E, electric field intensity; I, current; μ_1, first root of zeroth order Bessel function of the first king; δ, flow expansion coefficient ahead of arc; T, temperature; $F = (\delta\rho_0 h_1)^2/4\bar{N}\bar{\sigma}$; $E^2_{min} = Fv_0^2$; $\bar{h} = (h-h_1)/h_1$; J_0, J_1, zeroeth and first-order Bessel functions of the first kind; I_0, modified zeroth-order Bessel function; $w = h_1\rho v/\bar{M}$; $a^2 = \bar{N}/\bar{M}$. Subscripts: 0, gas parameters far from arc; 1, parameters at edge of arc; x, y, projections on coordinate axes; a, arc.

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THERMODYNAMIC CALCULATIONS OF COMBUSTION PRODUCTS AT HIGH PRESSURES

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A procedure for the calculation of chemical equilibrium in combustion products at high pressures (and high temperatures) is developed on the basis of a model equation of state. The conditions are determined for validity of the ideal-solution and covolume-gas approximations.

Methods for the calculation of equilibria of gas mixtures at high pressures have been discussed at length in the literature. The problem arises often in the study of solid-fuel combustion [1], in organic synthesis technology [2], in geochemistry [3], and in other applications. The fundamental problem is to find a sufficiently universal equation of state that is effective for the mixture at high parameters (up to ~5000°K and 10^3-10^4 bar). Various approximations are used for thermodynamic calculations in this range of the parameters, e.g., the combination of the Lewis—Randall approximation with the virial equation of state [4], or the single-fluid approximation [5, 6], in which the mixture is replaced by a hypothetical substance that is equivalent in some respect. In particular, the application of the virial equation of state in such calculations encounters difficulties associated with the determination of the third-fourth cross virial coefficients of the mixture.

In the present article we propose a procedure for calculating the equilibrium composition of reacting gas mixtures at high parameters on the basis of the equation of state postulated in [7, 8]. This equation of state has the specific attribute that it describes the properties of a high-temperature gas over a wide range of parameters with the use of data only on the second virial coefficients. The well-known mixing rule

$$B_{mx} = \sum_{i,j} x_i x_j B_{ij} \quad (1)$$

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